

## A COMPUTATIONAL METHOD FOR GENERATING CONVEX MAPS USING THE MEDIAL AXIS TRANSFORM

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### Abstract

*Convex maps were first introduced by Hillier and Hanson in “The social logic of space”, and have since become a standard diagram of space syntax, particularly in the analysis of interior of buildings. Despite of their extensive use, the computer generation of convex maps turns out to be quite difficult. An algorithmic description of the process would afford an objective form of reproducing analysis results and the consistent application of the same method on a set of data; unfortunately the original guidelines to draw convex maps by hand have been shown to be impossible to translate into a formal description of the type necessary in a computer program.*

*Thus, rather than attempting a translation of the original procedure, we have looked at alternatives methods in the field of shape analysis. In particular we have studied a set of problems which deal with the decomposition of shapes into simpler parts and their organisation, and which are conceptually related to the convex map. We have accordingly developed a method for subdividing architectural plans into non overlapping, convex partitions that captures their most salient organisational features, based on the medial axis transform, a well known shape descriptor first proposed by H.Blum in 1967. Our method is based on adding the simpler convex regions defined through the segments and branching points of the medial axis according to different priorities, under the condition that these additions remain convex.*

*In space syntax the automatic production of convex spaces has often focused on their instrumentality in the calculation of axial maps, and has not have the convex map as its final objective. The method we have developed, in the other hand, produces convincing convex partitions and maps, which often coincide with those resulting from following the original hand-drawn method description. Its results can be used for representing the organisation of spaces at the level of simplicity and abstraction of the original convex partitions proposed by Hillier and Hanson, and to allow the study of their configuration through the application of different graph measures and visualisation techniques.*

*We have implemented our methods in C++. The effective calculation of the medial axis required the use of robust and reliable computational geometry libraries, and consequently we have based most of our geometric data-structures and algorithms in those provided by CGAL, a well tested and widely used library distributed under GPL/LGPL license. Besides CGAL our tests and demonstration programs also use a number of different libraries, such as Dime, for dxflib input and output or Qt, for GUI and interaction.*

**Keywords:** Architectural Education, Evidence Based Design, Wayfinding Cognition, Virtual Reality

**Theme:** Architectural Design and Practice

## 1. Introduction.

Convex maps were first introduced by Hillier and Hanson in “The social logic of space”(Hillier and Hanson 1984), and have since become a standard diagram of space syntax, particularly in the analysis of interior of buildings. Despite their extensive use, the computer generation of convex maps is a difficult task. The original guidelines to draw convex maps by hand have been shown to be impossible to translate into a formal description of the type necessary for a computer program (Penn A 1997, Peponis, et al. 1997). Algorithmic methods for generating convex maps are useful at a practical level, since a rigorous algorithmic description of the process affords a consistent application of the method and the reproducibility of results.

Considering the difficulties of translating into an algorithm the original procedure described by Hillier and Hanson, we have looked at alternatives methods for producing convex maps in the field of shape analysis. In particular we have studied a set of problems which deal with the decomposition of shapes into simpler parts and which are conceptually related to the convex map. Our method uses the medial axis transform (MAT) - a well known shape descriptor first proposed by H. Blum in 1967 (Blum 1967)- to subdivide architectural plans into non overlapping, convex partitions. It produces convincing convex partitions and maps, which often coincide with those resulting from following the original hand-drawn method. The implementation makes extensive use of the Computational Geometry Algorithms Library (CGAL), particularly of the concepts from the Segment Delaunay Graphs (Karavelas 2013).

## 2. Background:

In our study we have considered convex maps consisting of adjacent and non overlapping convex areas, which result from following a drawing process first outlined by Hillier and Hanson in “the Social Logic of Space”. Convex maps afford a geometrical generation of adjacency graphs, in which relevant spaces of buildings and urban spaces can be subject to social and perceptual theorisation through concepts such as co-visibility or co-presence. During the 1980's and 1990's considerable effort was made in the automation of the process originally described in “the Social Logic of Space” as a set of rules of thumb for drawing convex partitions by hand. This was seen to be of “theoretical as well as practical importance in that it would be hard to maintain that a methodology was objective unless it could be satisfactorily automated” (Penn A 1997). The scarcity of automatic methods for generating convex maps confirms that the translation of the initial description by Hillier and Hanson into an algorithm is far from straight forward.

The Subdivision of polygons into convex partitions is a basic problem in the field of computational geometry (Preparata and Shamos 1985), where the different approaches have been rigorously studied in relation to their computational complexity, an aspect central to the practical implementation of any algorithm. The subdivision of a polygon with holes, as we may find in most outlines of building plans, has been proven to belong to the NP Hard complexity class (Lingas 1982). Besides the manifest computational complexity of the problem, the focus of computational geometry is in the optimisation not only of the time complexity, but also of the type and number of partitions produced, and the standard methods only incidentally produce partitions that are relevant from a spatial analysis perspective.

In the other hand, the related field of of shape analysis has produced a number of approaches to the analysis of forms based on shape descriptors, which are closer to the intentions and methods of spatial analysis. Shape descriptors are geometrical transformations of a shape that

capture features of interest. Some are graph-based, and consider the topological relations of a shape; a category of these is constituted by different types of skeletons, that is, transformations of boundary polygons into graphs that retain some (or all) of the information in the boundary. In our research we have put forward a number of methods for generating convex partitions based on the medial axis, or more accurately, in the geometrically equivalent notion of the segment Voronoi and its dual, the segment Delaunay triangulation (Kirkpatrick 1979).

The medial axis transform was first proposed as a form of shape descriptor by Harry Blum (Blum 1967, Blum and Nagel 1978). Based on the medial axis, August, Siddiqi and Zucker proposed a shape decomposition technique (August, et al. 1999) and Tănase and Veltkamp another based on the related straight skeleton (Tanase and Veltkamp 2003). Our method is close to these approaches, and implements a form of convex partition based on the medial axis that resembles the outcomes of the paper and pencil method to draw convex maps of Hillier and Hanson.

The use of the medial axis in the analysis of space was initially suggested by March and Steadman in “the Geometry of Environment” (March and Steadman 1971, 194). Other prominent examples can be found in Van Tonder, Lyons et al. analyses of Japanese Zen Gardens (Van Tonder, et al. 2002) and Rana and Batty’s examples of using shape analysis in architecture (Rana and Batty 2004). Wen-Chieh Wang (Wang 2012) has also proposed the medial axis to generate convex maps, though these are not strictly convex, their main purpose being the production of axial maps, and not of permeability graphs.

### **3. Our method for generating convex partitions using the medial axis.**

#### **3.1 Basic Concepts and terminology.**

For brevity's sake we will assume a familiarity with the concepts of the medial axis, and we will only provide a brief introduction to some basic notions related to our method. For detailed descriptions of the basic concepts we suggest the original papers by Blum (Blum 1967, Blum and Nagel 1978); For an overview of the applications of the medial axis in different domains, we recommend the paper by Leymarie and Kimia (Leymarie and Kimia 2006); For a precise description of our particular implementation of the medial axis, we advise to look at the CGAL User and Reference Manual’s chapter on Segment Delaunay Graphs (Karavelas 2012). In the following text we also often use the words boundary and polygon interchangeably, since in our implementation building plans are defined using closed polygonal boundaries.

There are different ways of describing the medial axis (Figures 1, 2.A, 3.A and D, 4.A, 5 and 6.A ): put simply, the medial axis can be explained as the set of points that are equidistant to at least 2 points of the boundary of an object or shape. Blum proposed a grassfire analogy: if a fire would start around the boundary of a shape simultaneously, the medial axis would appear as the points in which different fronts of the fire meet (Blum 1967). Similarly the medial axis can be defined as the locus of the centres of maximal circles: that is, circles inside a shape that are not themselves enclosed in any other circle that is also inside the shape (O’Rourke 1994, 180). The medial axis can also be thought as a Voronoi diagram generalised to consider line segments as vertices (Kirkpatrick 1979).

If we consider the medial axis as the locus of the centres of circles touching the boundary in at least two points, Blum and Nagel (Blum and Nagel 1978) distinguished between *normal points*, which are centres to circles that touch the boundary in only two places, and *branch points*, which touch the boundary in more than 2 points. *Normal points* correspond to straight or

curved segments of the axis, and *branch points* correspond to points in the medial axis in which the axis separates into different branches. In figures 1 A and 5, branching points are the centres of the drawn circles.

Blum and Nagel make also a further distinction fundamental to our method, based on the relation of points in the medial axis with sharp corners on the boundary of the shape: following the nomenclature by August et al. (August, et al. 1999) , we call segments of the medial axis closest to one corner and a straight segment *semi-ligatures* (coloured red in our figures) ; the portions of the axis defined by the proximity of two reflex vertices are alternatively termed *full-ligatures* (coloured orange). If we consider the above mentioned polygonal boundaries to correspond to plans of buildings, we can think of the medial axis as a description of proximity relations: *semi-ligature* segments (red) indicate a relation of proximity of a corner and a wall; *full-ligature* segments (orange) relations of proximity of two corners; the rest (straight segments drawn in blue in the figures) relations of proximity of 2 walls. In *branch points* more than one of these relations exist simultaneously.

Another relevant concept to our method, introduced also by Blum and Nagel in their 1978 paper is that of a *symmetric chord*. A *symmetric chord* consist of the line joining the two points on opposite sides of the boundary closest to a point on the axis. Figure 1C shows some relevant *symmetric chords* of the medial axis of a simple shape. The symmetric chords of the first and last point of ligatures will in effect define the limits of the convex partitions of the method described in this paper.

### 3.2. Description of the method: using the medial axis to generate convex partitions.

Our method is reminiscent of the decomposition of August Siddiqi and Zucker (August, et al. 1999), as it also uses Blum and Nagel's *ligatures* (*semi-ligatures* and *full-ligatures*) to define boundaries between significant regions of a shape. However the need for these regions to be convex in our case, has lead to a different approach, in which first the shape is decomposed into strictly convex parts that are later additively composed into larger convex partitions. There are thus two basic aspects to our method: first, the definition of these initial convex subregions, and second, the rules of addition of these sub-regions into the final convex partitions.

The basic premise for initially defining convex regions is based on the fact that every connected portion of the medial axis which does not include any *ligature*, will define a convex polygonal region (Figure 1C). As a consequence, convexity in a polygonal boundary will only be broken around *ligatures*, that is, in the regions defined by the proximity of a corner or reflex point. This however, does not mean that *ligatures* will always be the limits of convex partitions. In Figure 1 and 2 the sub-regions consisting of straight segments (in blue) can still be joined along the *ligatures* to form larger convex regions. The union of these convex sub-regions imposes however a few algebraic constrains. Particularly the operation is not associative, that is, the sequence in which the unions are performed affects the results, as can be seen in Figure 2. Thus our method is based in ordering the *ligatures* and then testing the success or failure of joining its two adjacent convex regions. If some unions prevent others from happening (as in Figure 2) then it is important to decide which unions should have priority over others, that is, what type of convex spaces should be created first.

Following the definition of the medial axis as the centres of maximal circles (O'Rourke 1994), some of its *branch-points* will constitute the centres of circles that are locally maximal; a circle with centre at any of those local maxima *branch points* will effectively be the largest circle that

can fit in a certain part of a shape (plans of buildings in our case). This characteristic coincides with the method given by Hillier and Hanson, based on finding largest circles in the  $y$ -space (Hillier and Hanson 1984, 98). We have thus developed a heuristic based on this observation, which orders *ligatures* according the radius of the the smallest circle they define, from larger to smaller. When the algorithm tests the potential unions along the ordered *ligatures*, it effectively pushes the places in which convexity will be broken away from the largest circles, and into the narrowest parts defined by the boundary. The convex partitions will be made of portions of the boundaries with largest possible tangent circles.

### 3.3. Variants of the ordering: emphasising the effects of bottlenecks, corners, and symmetry in the convex partition of a plan.

The basic heuristic based on ordering *ligatures* according the radius of the the smallest circle, as outlined above, produces results in which larger convex spaces will take priority over smaller ones (Figure 3.E). Depending of the type of analysis, this may not always be desired. The use of the medial axis allows to establish different ordering priorities, to account for different shape features of building plans: We may consider that the corner-wall relation of a *semi-ligature* should take precedence over the corner-corner “bottlenecks” produced by *full-ligatures*, rather than giving them equal priority (Figure 3.F). We may also assess that convex corners are shape features that should take precedence over the size of the radius of tangent circles (Figure 3. G) in the process of generating convex partitions; we may account for this by processing first the branches connected to *terminal points* (Blum and Nagel 1978) and then the rest of the medial axis.

A problem highlighted in other efforts to algorithmically generate convex maps is that arising when the criteria of Hillier and Hanson’s description of having fewest and fattest convex spaces may be fulfilled identically by a number of different partitions (Penn A 1997). We find a related problem in our method when various *ligatures* with the same priority can independently join adjacent convex spaces, but their simultaneous union into a single convex space is not possible. In our method we have called this problem *concurrency*. A typical example is that of a simple cross shape (Figure 3.A, 3.B and 3.C). The basic convex-subspaces to join can be seen in figure 3.A . It is easy to see that the four unions along the *full-ligatures* of each of the arms with the centre are possible independently, and even two opposite arms can be joined simultaneously (Figure 3B) but not all four unions are possible at the same time. The basic algorithm proposed above will consider each case independently in an arbitrary order, and join as many as it can (Figure 3.B). This may in simple cases provide a satisfactory arbitrary solution. However, it does not provide a general, reproducible form of dealing with the problem, and the results of the arbitrary ordering, in highly symmetrical shapes, are far from desirable (Figure 4.B). Our solution to this *concurrency* problem is to only perform unions that are not dependent of other *concurrent ligatures*; thus in the case of unions being inter-dependent, the unions won’t be performed (Figure 3.C, 4.C and 4.D). This is an important departure from the original pen-and paper method; we compromise the production of fewest convex partitions when this may imply arbitrary decisions.

### 3.4. Filtering of convex spaces based on shape cues.

The processes described so far produce strict convex partitions of the space. These partitions will in have an associated graph with convex partitions as nodes, and their relations of adjacency as edges (Figures 6.B and 6.C). However, some of the resulting convex spaces may not be relevant for a particular analysis of the space. This may be the case of door thresholds or small residual spaces in the plan. It is possible to use the information of the medial axis to

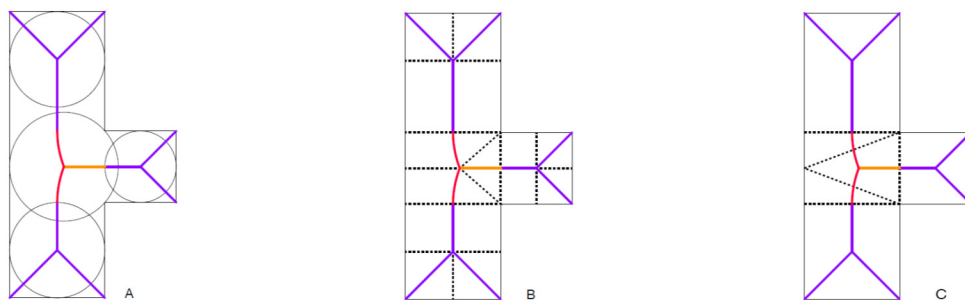
analyse and identify these cases Accordingly, we have implemented 3 filters to simplify the final convex map by eliminating isolated corners, spaces defined only by wall thicknesses (such as door thresholds), and small residual spaces. The results of this filtering process can be seen in figure 6, which shows our method applied to the plan of Luis Sullivan’s Peoples Saving Bank at Cedar Rapids, Iowa. Figure 6.A shows the medial axis, and 6.B the convex maps resulting from applying our method. Figure 6.C shows this convex map directly translated into a graph, and 6.D. The graph after simplification through filtering.

### 3.5. Some details about the implementation.

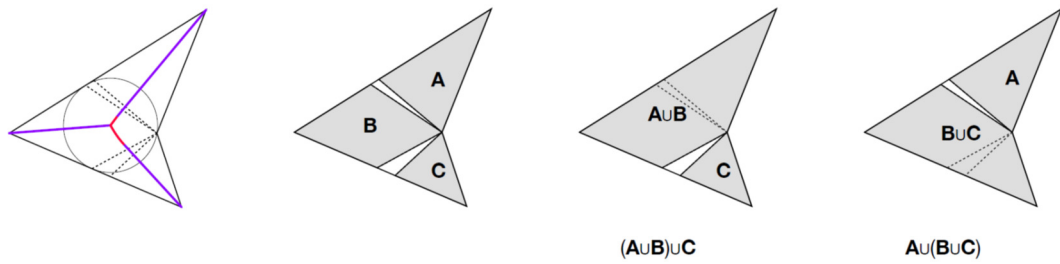
The complexity of the medial axis algorithm used, that is, the Segment Delaunay Graph from CGAL, is  $O(n \log^2 n)$  for polygons, where  $n$  is the number of segments of the polygons (Karavelas 2004). The complexity of our method is dependant of the implementation of the sorting used for the *ligatures* of the medial axis, and the number of these. We currently use the C++ stl *priority\_queue*, which has a default complexity of  $O(n \log n)$  for insertions, where  $n$  is the number of *ligatures* to be processed, and which does not substantially affect the upper bound of the complexity given by the CGAL Segment Delaunay calculation.

### 4. Remarks and Conclusions.

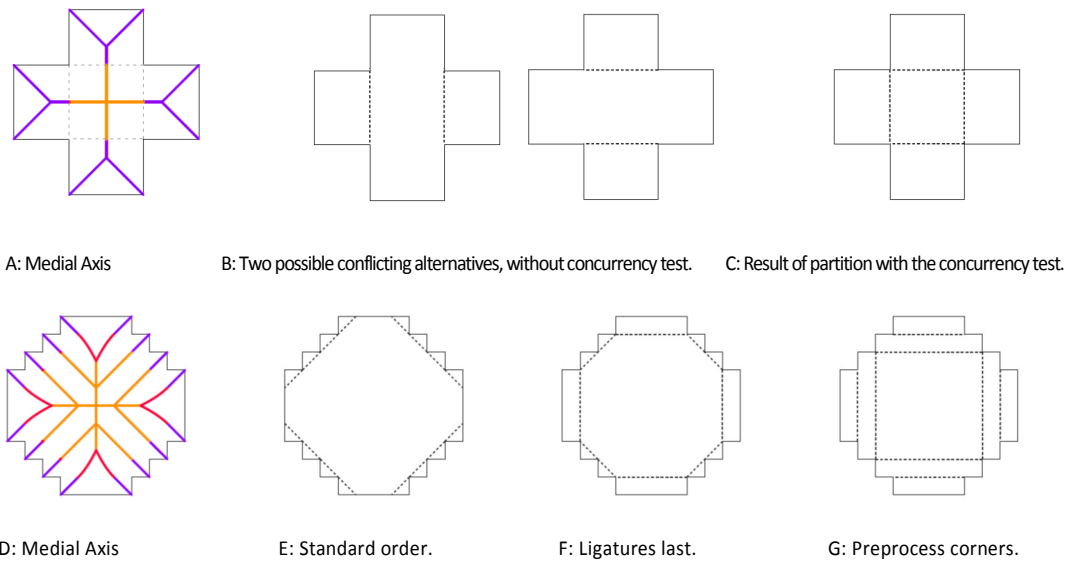
Our method differs, sometimes substantially, from the original process description by Hillier and Hanson; our purpose has been to produce a convex map with graph properties similar to those made by hand, and convex partitions that coincide generally, even if produced differently, with the convex partitions of the original heuristic. The results of our methods, applied to for example the classical small town of Gussin from “the Social Logic of Space”, are very similar to the original, as it is apparent in figure 5, with very small differences also between the diverse versions of the method. Rather than providing one single solution for the generation of convex maps our intention has been to propose and demonstrate a number of strategies for creating convex partitions that can be consistently reproduced.



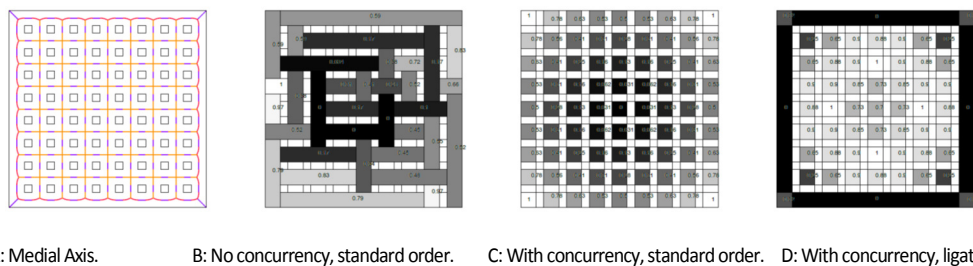
**Figure 1:** The medial axis of a simple polygon, with straight segments of the axis in blue, semi-ligatures (parabolic sections) in red, and full ligatures in orange. Figure 1 A shows the circles touching to more than 2 boundary points (centred at the branching points). Figure 1 B shows the partition into simplified segments. Figure 1 C shows the partition along symmetric chords of full-ligatures and semi-ligatures, used in our method.



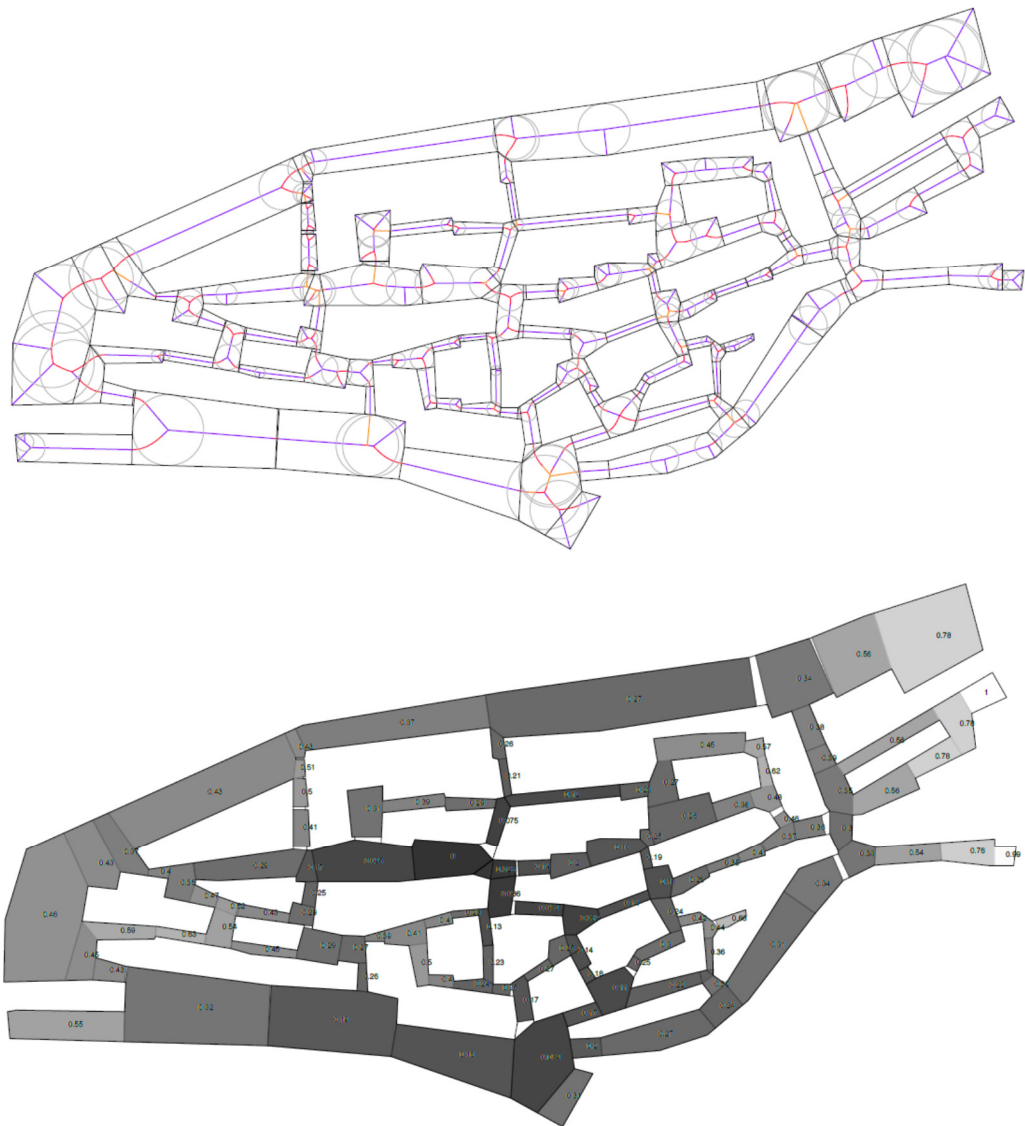
**Figure 2:** Medial axis of a simple form and partitions along symmetric chords. The figure illustrates the lack of the associative property for the union of partitions, and the reason to base our method in the order in which these unions take place: If A, B and C are adjacent convex spaces,  $A \cup B$  and  $B \cup C$  may be both convex, but not  $A \cup B \cup C$ . Thus  $(A \cup B) \cup C$  will produce a different result than  $A \cup (B \cup C)$ : The first case will result in the 2 convex spaces  $(A \cup B)$  and C, and the second case in the convex spaces A and  $(B \cup C)$ . The operations are thus commutative ( $A \cup B == B \cup A$ ,  $C \cup B == B \cup C$ ,  $(A \cup B) \cup C = C \cup (A \cup B)$ ...etc) but not associative.



**Figure 3:** Examples of different ordering sequences of unions along ligatures: A,B and C: The case of an ambivalent cross plan (all sides are equal width): solved in the first case by our method arbitrarily (B) and using the concurrency test (C). D, E, F and G: Some results of treating features by the different sorting orders: E, standard order, F full-ligatures considered last, and G, preprocessing the corners of the boundary first.

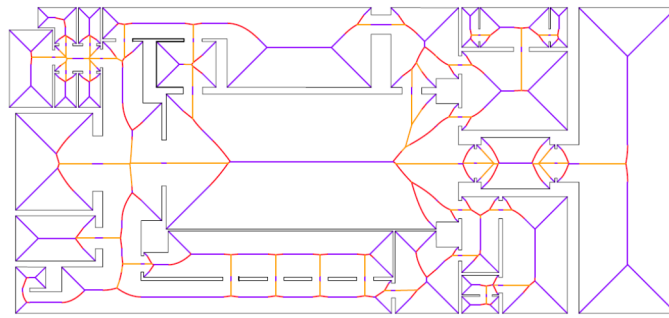


**Figure 4:** A grid case with a large number of concurrent edges, and the results of applying different priority ordering schemes. The resulting graph has been used to calculate relative asymmetry, displayed in a white to black scale (high asymmetry lighter, lower asymmetry darker) Observe the difference in results of using different partition schemes, from the arbitrary results of the standard process without concurrency test (B) or the regular concurrency test (C) to the effect of using both concurrency test and processing the full-ligatures last, which registers the differences on the edge of the grid (D).

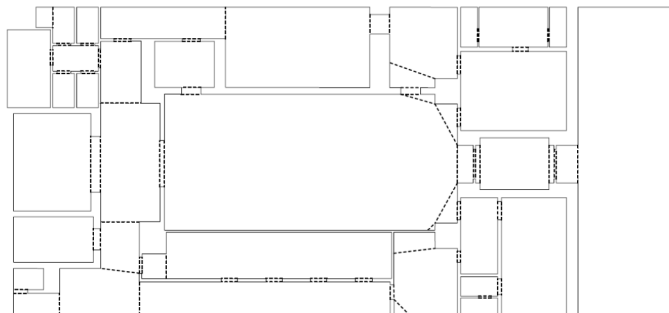


**Figure 5:** The small village of G, used in “the Social Logic of Space” analysed through our method. On top is the medial axis and all circles centred in branching points (points in the axis with contact with the boundary in more than 2 places). As previously, blue segments show parts of the medial axis produced by 2 straight segments in the boundary, red indicates intervals in the axis generated by the adjacency of a reflex vertex and a straight segment (semi-ligatures), and orange those produced by two reflex points (full-ligatures). The bottom figure shows the result of applying one of our methods (corners first and no concurrency, though the different methods make very little difference in this case), and the calculation of relative asymmetry on the resulting graph. Some small white gaps in the graph belong to small residual areas that have been filtered out, as described in the paper. Compared with the original hand drawn convex map, the difference are very small. In some cases these are produced by imprecisions in the drawing of the original map, other times because some of the aspects of the method, like “fatness” seem to have been ignored in the original drawings in “the Social Logic of Space. It is interesting to see that the degree variation of the connections, at least in this case, is small (most spaces have a degree 2, crossings and small squares, degrees of 3 or 4), particularly compared with the different results in buildings (see figure 6).

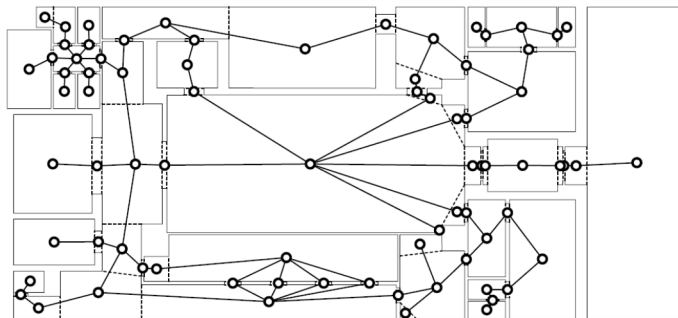




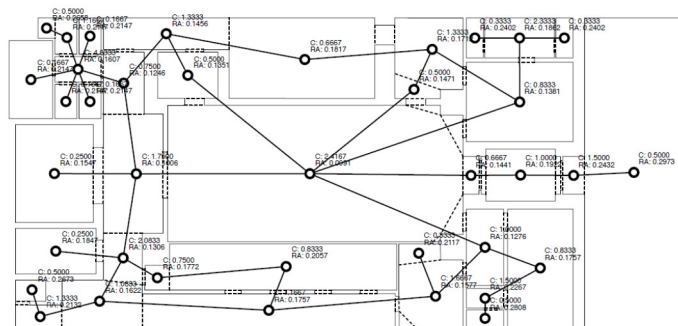
A. Medial axis. Blue indicates that the two boundary sides on the interval are line segments. Red indicates an interval generated by a reflex vertex and a line segment (semi-ligature). Orange, by two points (full-ligature).



B. Convex subdivision result of the algorithm applied on the medial axis.



C. Graph of all convex spaces, before applying filtering algorithms.



D. Graph (convex map) after filtering thresholds and corners, showing control and relative asymmetry measures.

**Figure 6:** The standard method applied to the plan of Luis Sullivan's Peoples Saving Bank, Cedar Rapids, Iowa, 1911. In Figures C and D it is possible to see the effect of the filtering of the graph: figure C shows the graph of the convex map resulting from the strict geometric definitions. Figure D shows the graph after filtering using cues from the medial axis: Doors, small thresholds and residual spaces result of misalignments and small perturbations on the boundary can be identified and dealt with. Observe also the obvious difference in the degree distribution, compared with the analysis in figure 5 of the village of G.

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