

SPACE SYNTAX ANGULAR BETWEENNESS CENTRALITY REVISITED

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Abstract

The main objective of the paper is to revisit the different implementations of angular betweenness in terms of calculation, efficiency and accuracy. The idea is to compare angular betweenness using graph-search-agent borrowed from network science theory and cognitive-search-agent which dominates the space syntax research through the use of depthmapX (and Depthmap in the past). The study uses the dataset from Penn et al. (1998) and Hillier and Iida (2005), for comparison and validation, where the new angular betweenness centrality proposed, that derives from a graph-search agent idea, performs faster with similar explanatory power on pedestrian movement than the space syntax depthmapX implementation which uses cognitive-search-agent. Finally the findings, which show that a graph-search agent may be better relate to manifestations of spatial perception, such as way-finding and pedestrian movement, reveals the need for further exploration of the underlying similarities and differences of angular betweenness using graph-search agent and cognitive-search-agent in relation to observed pedestrian behaviour.

Keywords: Network science, space syntax, betweenness centrality, angular choice

Theme: Modelling and Methodology Developments

1.0 INTRODUCTION

An important measure in space syntax research is ‘angular segment choice’ also known as ‘angular shortest path betweenness’ in network science where the measure have found strong correspondence in explaining aggregate pedestrian movement and vehicular movement (Penn et al 1998; Hillier and Iida 2005). The main objective of the paper is to revisit these graph algorithms by comparing the implementations of angular betweenness in space syntax and network science with the intention of optimising efficiency in terms of computation and accuracy in terms of correlation with observed movement.

In this paper, we propose to compare the implementation of angular betweenness centrality using two application platforms. The first application is ‘depthmapX’, an open-source multi-platform spatial network analysis software developed by Tasos Varoudis (Varoudis 2012) that forked the original ‘UCL Depthmap’ written by Alasdair Turner. The second application reviewed in this paper is an implementation of angular betweenness centrality written by Tasos Varoudis adapting from Brande’s weighted betweenness centrality algorithm (Brande 2001) and Dijkstra’s algorithm of weighted shortest paths. (Dijkstra 1949) This implementation will be called Tasos Implementation for the purpose of the paper only and the paper will use the term ‘angular betweenness’ for both implementations.

This paper is structured into five parts. First, we introduce the angular betweenness centrality measure within the context of graph theory and space syntax. Second, we describe the measure implementation using two platforms, depthmapX, and an implementation of angular betweenness centrality using the Tasos implementation. Third, we compare depthmapX and Tasos’ implementation results through statistical analysis in terms of betweenness centrality values and calculation time. Fourth, we will correlate the three sets of data to pedestrian movement data that was used from the paper of Penn, Hillier, Xu. (1998) Finally, we will summarise these results and suggests some steps forward. We will also discuss crucial questions that arise from this study and possible solutions that will close the cycle back to the justified graph.

2.0 BETWEENNESS CENTRALITY

Network science is an interdisciplinary academic field studying the complex relationships between related components using graph theory which has been applied in social, biological and transportation networks. The idea of centrality in social networks were first applied to human communication by Bavelas (1972) who was interested in how groups of people communicated and studying the relationship between centrality and influences. Since then, measures of centrality have been proposed by Freeman (1977) to quantify the importance of an individual in a social network. Betweenness centrality proposed by Freeman in 1977 is one such measures that has been used consistently in studying spatial and aspatial networks. Betweenness centrality calculates the number of shortest paths overlap from all origins and destinations. High values of centrality indicate a node lies on the shortest paths for high number of nodes in the system. If one removes the node with high betweenness centrality, it will lengthen the paths between many paths of nodes where in the extreme case, you disconnect an entire system by removing the only bridge. Betweenness centrality is expressed more formally below, where $g_{jk}(p_i)$ is the number of geodesics between node p_j and p_k which contain node p_i and g_{jk} is the number of all geodesics between p_j and p_k .

$$C_B(P_i) = \sum_j \sum_k g_{jk}(p_i) / g_{jk} (j < k)$$

1. Betweenness Centrality from Hillier and Iida (2005) referring to Freeman(1977)

2.1 Space syntax segment angular choice

Space syntax is a set of theories and techniques which applies graph measures to study the configuration of spatial networks in architecture, urban design and transport planning. It is based on research by Bill Hillier and Julienne Hanson and their colleagues at University College London (Hillier, Hanson 1984). In a space syntax network model, each street is drawn to represent the longest lines of sight between all connected convex spaces, known as the axial line (Hillier 1996). Recent developments consider the use of road centre lines to create the spatial network model in computing centrality (Turner 2007). One of the most important propositions in space syntax theory is the relationship between space and movement (Hillier et al. 1993). Empirically, space syntax research has continuously found a strong relationship between graph measures with pedestrian flows and vehicular flows (Penn et al. 1998).

Space syntax uses a similar measure to Freeman’s weighted betweenness measure (formula 1) as mentioned in Hillier and Iida’s paper. Different from Freeman’s roots in social networks, space syntax choice measure emerged from its architectural roots first presented in ‘Creating Life: or does architecture determine anything?’ (Hillier et al 1987) the concept of the justified graph as illustrated in the figure below and lastly angular analysis first introduced from Turner (Turner 2000) and Dalton (Dalton 2001) which accumulated into Hillier’s work in 2005 in comparing angular weighted betweenness, topological weighted betweenness and metric weighted betweenness. (Hillier and Iida 2005) A more formal description can be seen from Hillier et al.’s paper on normalizing angle choice. (Hillier et al. 2012)

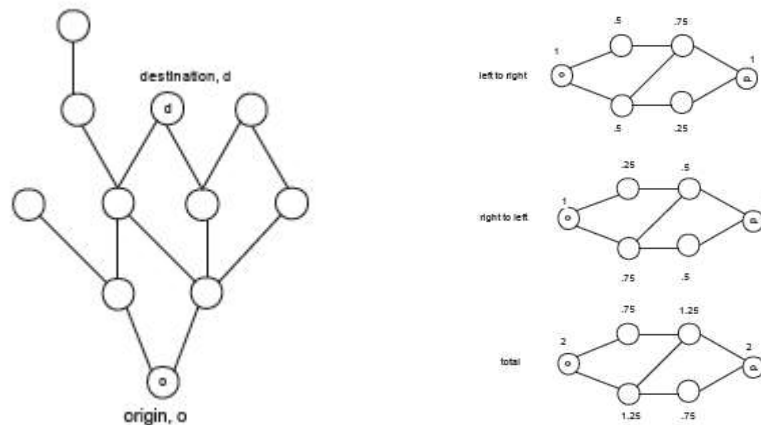


Figure 1: (Left) Justified graph between origin and destination pair (Hillier et al 2012) (Right) Distributing centrality values according to choices (Hillier et al 2012)

An examination and comparison of the different implementations of betweenness centrality in efficiency, accuracy and explanatory power on pedestrian movement will follow. The image below illustrate angular segment choice for Tokyo.

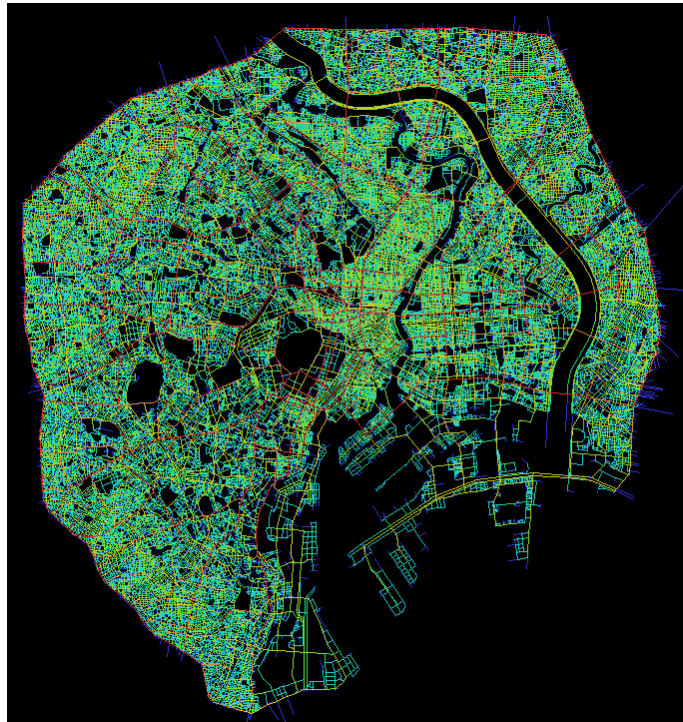


Figure 2: Tokyo angular betweenness centrality

3.0 BETWEENNESS IMPLEMENTATION COMPARISON

Angular betweenness also known as segment angular choice in space syntax literature calculates the number of shortest paths overlap between all nodes in the graph and is implemented in depthmapX. The figure below illustrates the shortest path search where you first draw a spatial network graph, then you convert it into space syntax network graph where the segment is the node and the junction is the edge. The space syntax representation of the graph differs fundamentally from traditional spatial network analysis. After this you identify a set of origins and destination and calculate the weights between the pair of origin and destination. Finally you draw the shortest paths based on these weights.

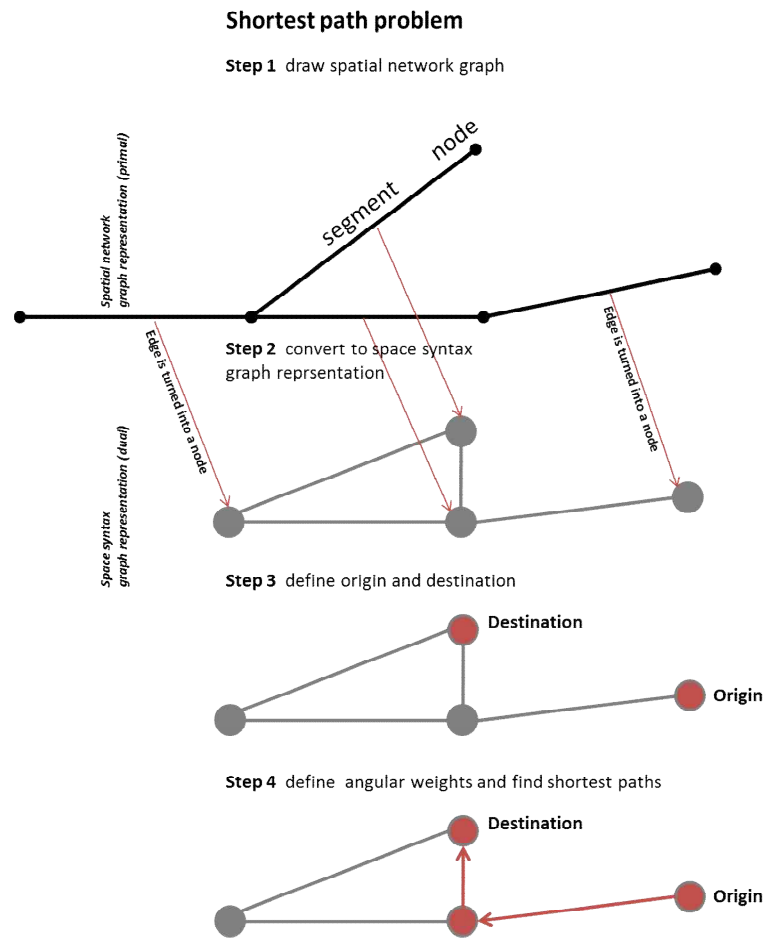
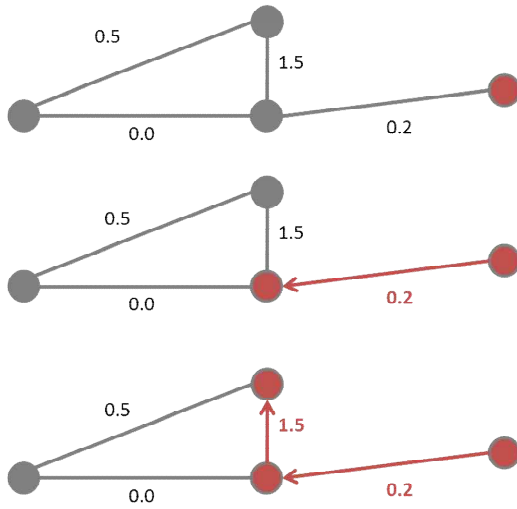


Figure 3

- Step 1 Draw spatial network graph
- Step 2 Convert to space syntax graph representation
- Step 3 Defining origins and destination pairs
- Step 4 Defining weights between origin and destination pairs and identify shortest path

This research proposes an alternative implementation of angular betweenness here named Tasos implementation that uses a different search algorithm when identifying shortest paths in comparison to depthmapX. The key difference between the two search implementations is that depthmapX allocates a pedestrian walk constraint at the junction of the search which is not necessarily the graph's shortest paths. In contrast, Tasos implementation does not allocate a pedestrian walk constraint in finding the true shortest graph paths and follows a mathematical definition of the shortest path. The figure below illustrates a case where the mathematical shortest path between A to B will pass through C in Tasos implementation but will not pass through C in depthmapX definition. We termed depthmapX implementation of the shortest path as a cognitive-search-path whilst we termed Tasos mathematical implementation of the shortest path as a graph-search-path to differentiate the two.

Step 4A cognitive shortest path
depthmapX implementation



Step 4B Graph shortest path
Tasos implementation

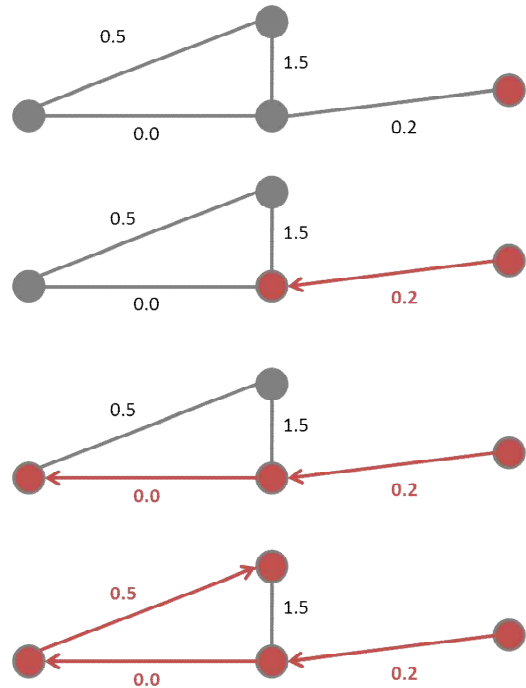


Figure 4
Left DepthmapX implementation
Right Tasos implementation

A more detail description of the two implementations and differences are described below.

3.1 depthmapX

DepthmapX' is an open-source multi-platform spatial network analysis software developed by Tasos Varoudis (Varoudis 2012) as a fork of the original Depthmap, first written in 1998 by Alasdair Turner and further developed by Eva Friedrich and Tasos Varoudis. The software was written for the purpose of measuring isovists and calculating graphs algorithms. It is currently the most popular software in calculating space syntax related measures with an estimated number of more than five thousands users. The software has been used to calculate graph as large as one million edges. The software has become open source since 2011 under the GPLv3 open source license. DepthmapX calculates angular betweenness using a cognitive-graph-search-path as suggested in the previous Section where the shortest route calculation in this implementation makes use of the pedestrian walk constraint. Furthermore, depthmapX implementation is a probabilistic one and uses a random subset of origin-destination pairs with bi-directional values in calculating angular betweenness.(choice).

3.2 Tasos implementation

This research proposes to re-evaluate the use of fundamental graph theory algorithms that are commonly use in network science for use in analysing space and movement. While depthmapX uses cognitive-graph-search-paths agents that have a global sense of the map and are restricted by spatial features (e.x. movement at junctions), Tasos implementation removes this pedestrian walk constraint restriction on spatial features and uses a mathematical-graph-search agents that searches through the shortest graph path within a mathematical representation of a graph.

Removing the restriction goes back to the roots of the mathematical betweenness and in relation to the justified graph in space syntax.

This implementation has three main features that distinguish it from depthmapX. Firstly, it calculates betweenness for all origins and destinations pairs (of segments – graph vertices) in one direction and not a selection of them for bi-direction. Secondly the new implementation uses precise angular edge weights based on the degrees between the segments rather than quantising angles into a number of bins as calculated in depthmapX. Thirdly, the ‘agent’ that searches through the graph for the shortest paths is only aware of the fundamental graph representation (vertices with weighted edges between them) and its not restricted by the notion of pedestrian walk constraint. This paper suggests that the proposed alternative implementation of betweenness centrality weighted by angular change achieves a faster completion time but with similar or better correlation to pedestrian movement compared to depthmapX. It should also be noted that in this paper we will focus only on the comparison between depthmapX and Tasos implementation of angular betweenness. Initial research suggests Pajek was significantly slower in processing the same graph for topological analysis as compared to depthmapX. Future research should include further comparisons with other implementations, such as Segmen, a software written by Shinichi Iida for calculating angular betweenness(Iida 2000)

4.0 THE CASE STUDIES AND COMPARATIVE ANALYSIS

In order to compare efficiency and accuracy between depthmapX and Tasos implementation of angular betweenness, the case studies of Barnsbury, Clerkenwell and Kensington are examined. The dataset for the case studies comes from Alan Penn’s study in 1998 (Penn et al. 1998) and has been subsequently used by Bill Hillier’s study in 2005 (Hillier and Iida 2005). In the first part of the comparative analysis, the three case studies are introduced. In the second part, the two implementations will be compared through a series of scatterplots examining correlation coefficients (R-square) between the betweenness measures and pedestrian movement. In these scattergrams we will plot the logged value of ‘choice’ obtained from depthmapX (LogCh) against the logged value of angular betweenness obtained from Tasos implementation (LogTasos)

In the third part, the computer calculation time for the two implementations of angular betweenness are compared. Computer calculation time here is defined as the time it needs to process angular betweenness calculation with the same computer for the same graph with the two implementation definition as explained in the last section.

In the fourth part of the comparative analysis, pedestrian movement observed in the three areas are plotted against angular betweenness in a scatterplot and correlation coefficients (R-squared) are compared. In this analysis we will plot the logged value of pedestrian movement (LogPedmov) against the logged value of choice (LogCH) obtained from depthmapX and the logged angular betweenness from Tasos implementation (LogTasos)

In this part of the study we will use an approach set out from previous research (Hillier and Iida, 2005) to carry out regression analysis between different implementations of angular betweenness and pedestrian movement. This method has been used to compare the differences between metric, topological and angular betweenness centrality with pedestrian movement.

4.1 Descriptive Statistics

This paper uses the same pedestrian movement dataset as in Penn’s paper entitled, Configurational modelling of urban movement networks (Penn et al. 1998) and in Hillier’s paper entitled Network and psychological effects in urban movement (Hillier and Iida 2005)

4.1.1 Barnsbury

Barnsbury is located to the north of Kings Cross Station in Central London with 7,269 segments. The map below illustrates the spatial network in red and the pedestrian movement gates number in black. The average, median, standard deviation and maximum angular betweenness values produced by depthmapX are larger than Tasos Implementation. The difference is attributed to the approximation of values in depthmapX calculation of angular betweenness as explained in the random selection of origin and destination pair and depthmapX calculates bi-directional values for each shortest path.

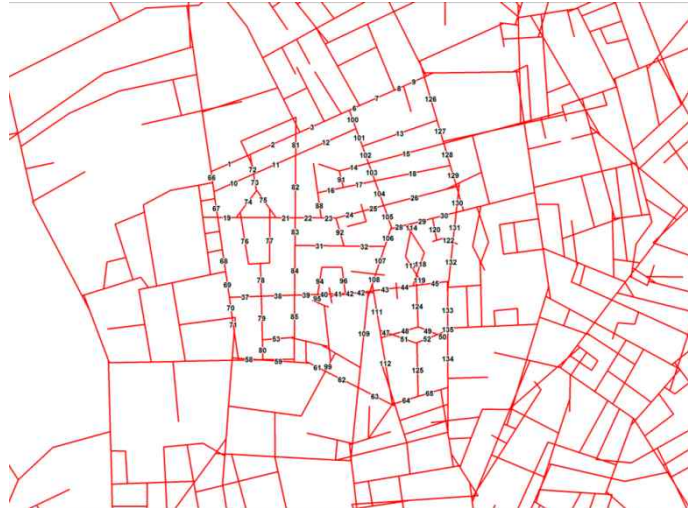


Figure 5: Barnsbury spatial network and pedestrian gate locations

Barnsbury	Choice (depthmapX)	Betweenness (Tasos Implementation)
N	7269	7269
Average	343359	210030
Median	41033	29197
Maximum	5971684	3514534
Minimum	0	0
Standard Deviation	821411	482822

Table 1: Barnsbury angular betweenness values

4.1.2 Clerkenwell

Clerkenwell is located to the south-east of Kings Cross Station in Central London with 14,853 segments. Similarly depthmapX calculates higher average, median, standard deviation and maximum angular betweenness values than Tasos implementation. The map below illustrates the spatial network in blue and the pedestrian movement gates number in black.

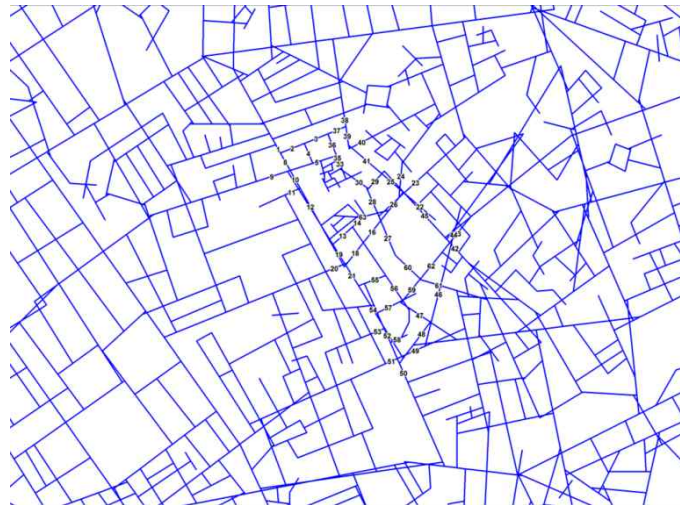


Figure 6: Clerkenwell spatial network and pedestrian gate locations

Clerkenwell	Choice (depthmapX)	Betweenness (Tasos Implementation)
N	14853	14853
Average	958230	564054
Median	80180	57700
Maximum	2.48 e+7	1.33 e+7
Minimum	0	0
Standard Deviation	2686112	1552620

Table 2: Clerkenwell angular betweenness values

4.1.3 Kensington

Kensington is located to the south of Hyde Park in West London with 14,853 segments. Similarly depthmapX calculates higher average, median, standard deviation and maximum angular betweenness values than Tasos implementation. The map below illustrates the spatial network in green and the pedestrian movement gates number in black.

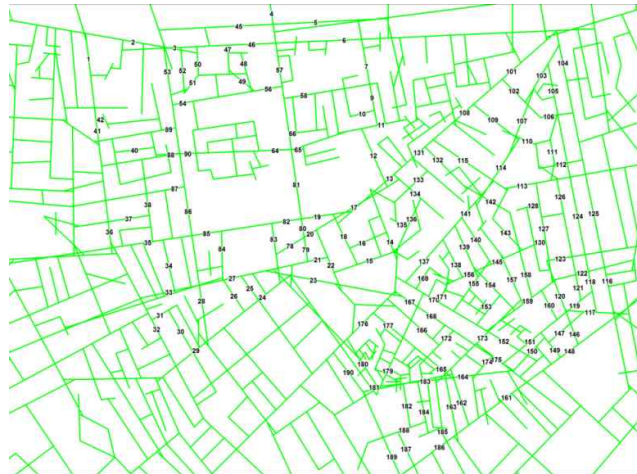


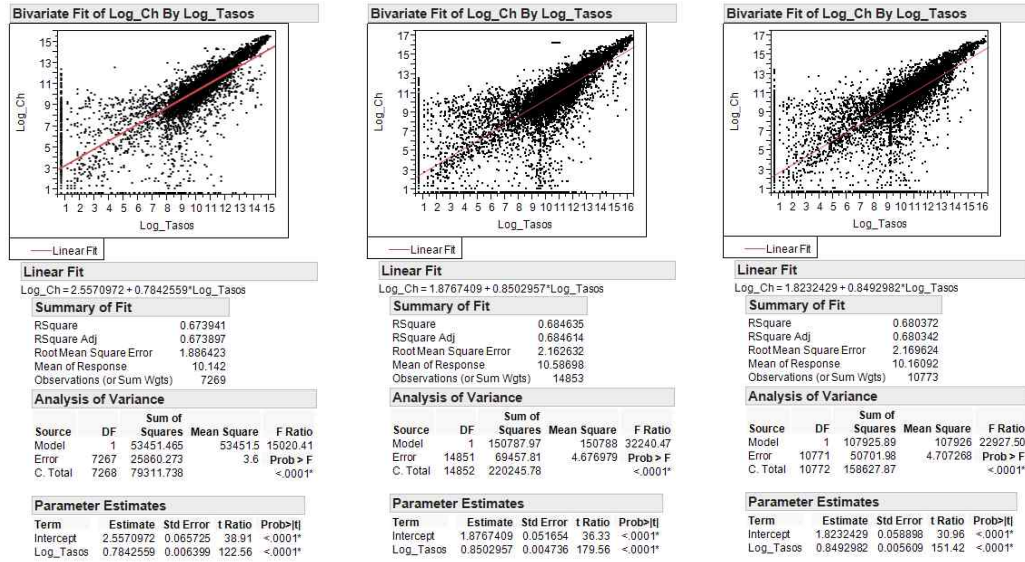
Figure 7: Kensington spatial network and pedestrian gate locations

Kensington	Choice (depthmapX)	Betweenness (Tasos Implementation)
N	10773	10773
Average	642510	365514
Median	55124	39667
Maximum	2.31 e+7	1.23 e+7
Minimum	0	0
Standard Deviation	1863566	1088173

Table 3: Kensington angular betweenness values

4.2 Comparative correspondence between values calculated by two implementations

In order to examine the relative differences between the two implementations, the plots below summarise the correlation coefficient between depthmapX logged choice values and Tasos implementation's logged angular betweenness values.



- 2 Logged Choice as calculated in depthmapX
 $LogCh = Log(T1024Choice+2)$
- 3 Logged betweenness as calculated in Tasos implementation in C++
 $LogTasos = Log(angular_centrality+2)$

Table 4: Regression results comparing depthmapX and Tasos implementation

The scattergrams suggest a generally positive correlation between the two measures for all three areas when comparing (R-squared value is approximately 68%). The results also suggest that above the mid-point there is higher correlation than below the mid-point where the data spreads more randomly in both x and y directions. These distributions suggest that there are marked differences between the two implementations. These differences can be attributed to the fact that depthmapX implementation of choice uses only a subset of the origins-destination pair whilst the Tasos implementation uses the entire set. The differences can also be attributed to differences in shortest path calculation in the use of full angular analysis instead of approximation using angle quantisation bins. Finally, the variations between the two measures can be attributed to the differences between how the shortest cognitive paths and shortest graph paths are calculated in each implementation.

For a better comparison between the two implementations, the measures of angular betweenness have been visualised for the three areas of London. Conforming to existing space syntax standards, higher centrality values have been visualised in warm colours (red, orange), while lower centrality values have been visualised in cold colours (green, blue). To make the maps comparable, a 16 colour natural break range have been used. The results suggest very similar spatial distribution of centrality values for all three areas specially in the red areas and blue areas. There are some isolated spatial differences in the mid-ranges but the general spatial structure and ranking remains consistent.



Figure 8
Top Barnsbury
Middle Clerkenwell
Bottom Kensington

4.3 Efficiency comparison

In this section we compare the efficiency of processing or computation time between the two implementations. For measuring the computation time an identical computer (an iMac 2011 3.2Ghz) has been used to run both implementation in identical conditions. The chart below illustrate the calculation time for processing angular betweenness comparing depthmapX and Tasos implementation for three cities. In general Tasos implementation is at least four times faster than depthmapX for Athens and Tokyo. The Greater South East cannot load in depthmapX but Tasos implementation can load the graph of 1.5 million vertices and calculating betweenness centrality for the entire graph with only one CPU-core in 15 days.

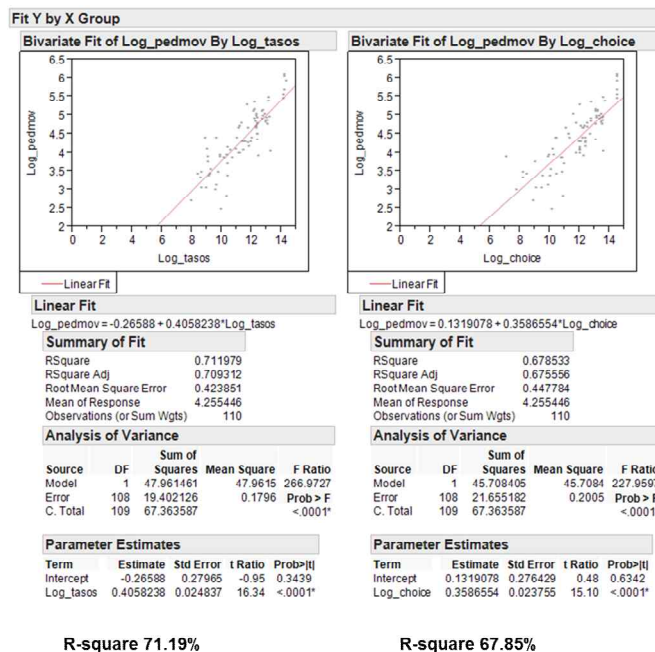
	Node	depthmapX	Tasos implementation	Differences (forced single core)
Athens	150,000	11 hours	2.7 hours	4 times faster
Tokyo	250,000	40 hours	9 hours	more than 4 times faster
Greater Southeast	1,500,000+	Cannot compute	15 days	

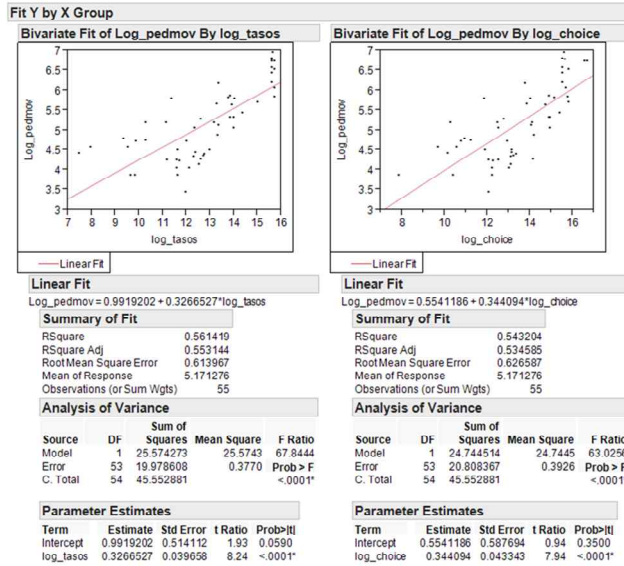
Table 5: Calculation time comparison

These results suggest a significant improvement in computer calculation time for the Tasos implementation. The results also suggest greater marginal benefits in efficiency as the size of the graph increases. In other words, the Tasos implementation becomes relatively faster when the size of the system increases.

4.4 Pedestrian movement correlation comparison

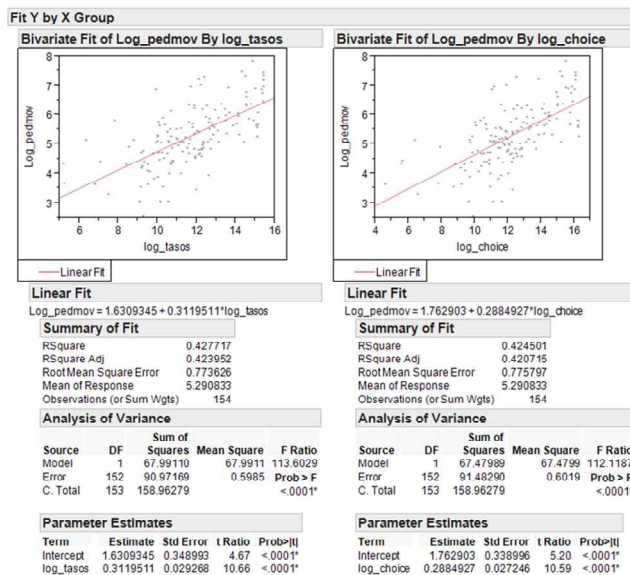
In the fourth part of our comparative analysis, we compare the values of correlation coefficients between pedestrian movement correlation and depthmapX and Tasos implementations in accordance to Hillier and Iida's approach (2005). The plot between logged pedestrian movement (LogPedmov₄) and logged choice as calculated in depthmapX (LogCh₂) and logged tasos implementation of betweenness (LogTasos₃) are illustrated in the figure below for all three areas in London.





R-square 56.14%

R-square 54.32%



R-square 42.77%

R-square 42.45%

- 2 Logged Choice as calculated in depthmapX
 $\text{LogCh} = \text{Log}(\text{T1024Choice} + 2)$
- 3 Logged betweenness as calculated in Tasos implementation in C++
 $\text{LogTasos} = \text{Log}(\text{angular_centrality} + 2)$
- 4 Logged pedestrian movement from Penn et al, 1998
 $\text{LogPedmov} = \text{Log}(\text{Pedestrian movement} + 2)$

Table 6:
Top Barnsbury
Middle Clerkenwell
Bottom Kensington

The graph below summarises the R-square between pedestrian movement with both depthmapX and Tasos implementation. While the results suggest highly similar scattergrams, best fit line, standard errors and test statistics for all three areas, there are slight increases in the values of correlation coefficients (between 0.75% to 4.9%).

R-square	depthmapX Choice	Tasos Implementation	Differences
Barnsbury	67.85%	71.19%	+4.92%
Clerkenwell	54.32%	56.14%	+3.35%
Kensington	42.45%	42.77%	+0.75%

Table 7: Pedestrian movement correlation comparison

5.0 CONCLUSIONS

This study has shown that Tasos implementation of angular betweenness, which uses a mathematical graph shortest path in its calculation without the pedestrian walk restriction in depthmapX, achieves a much faster computation time as compared to depthmapX implementation with better or similar accuracy in explaining pedestrian movement. This offers a great advantage for the analysis of large urban systems. Positive correlation in the scatterplot between the two implementations of angular betweenness suggests that the outputs are comparable. Since similar pedestrian movement correlations have been achieved across the three areas for both implementations, it can be concluded that both of these implementations are good indicators of pedestrian movement where up to 70% of the variation in pedestrian movement can be described by spatial configuration. This is an intriguing finding and counter-intuitive in terms of spatial cognition, as a mathematical-graph-search-path betweenness explains pedestrian movement similar to cognitive-search-paths betweenness. This might be an artefact of an urban system where both implementations achieve similar distribution but also the fact that the sample size does not necessarily reflect the differences between the two types of analysis. Further work is needed to verify this conclusion.

As the study has shown, if the time of computation is not an issue, both of these implementations offer very similar results in term of correlating with pedestrian movement. However, when the systems get very large (for instance +500,000 segments), and the computation time becomes an issue, the Tasos implementation seems to offer a significant benefit over the depthmapX implementation.

6.0 DISCUSSION AND WAY FORWARD

As a way forward, there is first a need to further develop Tasos implementation of calculating angular betweenness by incorporating some of the elemental principals captured in the justified graph, as opposed to the pure mathematical graph used for this paper. This will not only provide a more complete picture in understanding the trade-off between accuracy and performance but a link to research on spatial cognition and way finding in urban environments. Furthermore, it is essential to study the efficiency gains from multi-core processing of spatial network graphs exceeding 10,000,000 nodes. For this study Tasos implementation was forced to utilise a single-CPU-core in order to produce comparable results with depthmapX in terms of calculation times, in its core though it is developed so that multi-core processors can be utilised easily.

Lastly, the intriguing findings where a mathematical-graph-search-path betweenness explains pedestrian movement similar to cognitive-search-paths betweenness not only answers the original research objective of this paper in revisiting angular betweenness but more significantly

reveals the need to examine the differences between the two. The authors believe the significance of the results illustrate one of the initial arguments of the paper. A simply build graph, which encapsulates the fundamental properties of the spatial systems that it represents, presents a strong relation with pedestrian movement.

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